Exam Quantum Physics 2

Date	18 June 2014
Room	K. Duppenhal
Time	8:30 - 11:30
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- Raise your hand for more paper
- You are allowed to use the book "Introduction to Quantum Mechanics" by Griffiths
- You are *not* allowed to use print-outs, notes or other books
- The weights of the exercises are given below
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Weighting

1a)	7	2a)	10	3a)	8
1b)	10	2b)	7	3b)	10
1c)	10	2c)	7		
1d)	7	2d)	7		
		2e)	7		

$$\text{Result} = \frac{\sum \text{points}}{10} + 1$$

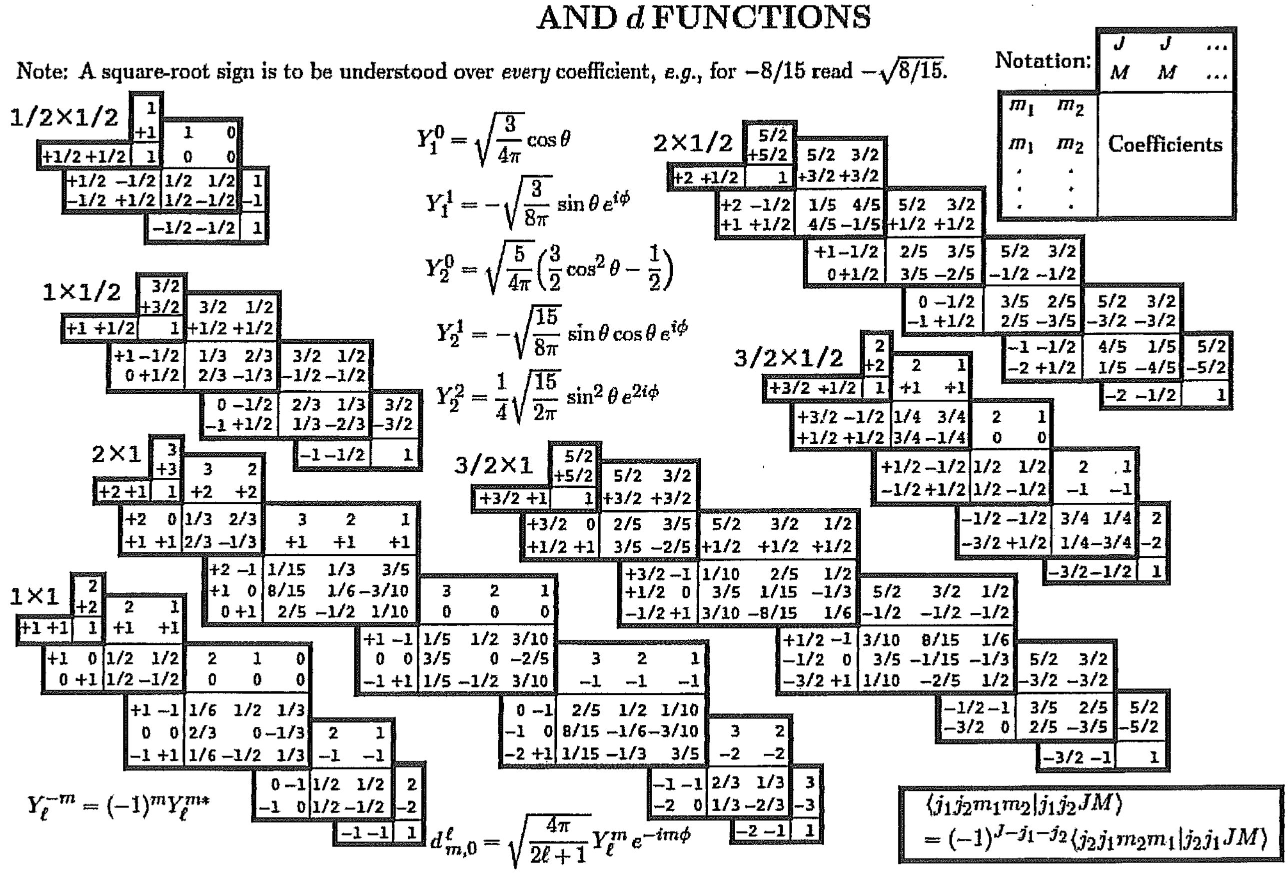
Exercise 1

(a) Explain what are the properties of operators that are constants of motion and how they are used to describe stationary states.

(b) Use the table below to write down the Clebsch-Gordan decomposition of the state $|l, s; j, m_j\rangle = |1, \frac{1}{2}; \frac{1}{2}, \frac{1}{2}\rangle$ and verify that acting with J_+ on the decomposition gives zero.

34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND *d* FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.



(c) In the calculation of the strong field Zeeman splitting, one can take into account the relativistic spin-orbit coupling effect as a perturbation. The first-order perturbative correction then involves the calculation of the matrix element $\langle l, s, m_l, m_s|\vec{L}\cdot\vec{S}\,|l, s, m_l, m_s\rangle$. Evaluate this matrix element and explain why one only has to consider such diagonal matrix elements, despite the degeneracy in some of the quantum numbers for the unperturbed system.

(d) Consider a Hamiltonian H that commutes with the parity or reflection operator P : $x \to -x$. Show that $\langle\psi_a|H|\psi_b\rangle = 0$ whenever ψ_a is an even function of x and ψ_b is an odd function. Explain how this result helps to simplify [redacted]degenerate perturbation theory calculations.

Exercise 2

Consider the one-dimensional harmonic oscillator as unperturbed system and introduce the perturbation

$$H'(x) = c\sqrt{b}\exp(-bx^2),$$

where b and c are positive constants.

(a) Calculate within perturbation theory the first-order correction to the ground state energy and determine for which values of c the result is valid when $b \gg m\omega/\hbar$.

Consider next the perturbation

$$H'(x) = c\,x\sqrt{b}\exp(-bx^2),$$

where b and c are positive constants.

(b) Show that in this case the first-order perturbative correction vanishes.

(c) Show that the second-order perturbative correction to the ground state energy is negative.

(d) Demonstrate using the variational principle that adding this perturbation H' can only decrease the energy of the ground state.

(e) Draw a picture of the potential including the perturbation H' and write down a trial wave function that might be expected to give a better upper bound on the ground state energy than the unperturbed ground state energy (motivate your choice).

Exercise 3

Consider the Hamiltonian $H = H_0 + H'(t)$, where H' is a time-dependent perturbation that is nonzero for $t \geq 0$. Let $\psi_n^{(0)}$ be the orthonormal set of eigenstates of H_0 with energies $E_n^{(0)}$, i.e. $H_0\,\psi_n^{(0)} = E_n^{(0)}\,\psi_n^{(0)}$.

(a) Show that with the following expansion on the states $\psi_n^{(0)}$

$$\psi(t) = \sum_n c_n(t)\,\psi_n^{(0)}\,e^{-i\,E_n^{(0)}t/\hbar},$$

the coefficients satisfy

$$\dot{c}_m(t) = \frac{1}{i\hbar}\sum_n c_n(t)\,e^{i\,(E_m^{(0)}-E_n^{(0)})t/\hbar}\,H'_{mn},$$

where $H'_{mn} = \langle\psi_m^{(0)}|H'|\psi_n^{(0)}\rangle$.

(b) Consider the case where $H'(t) = V(r)\theta(t)$ for a two-level system consisting of states ψ_1 and ψ_2, such that $\langle\psi_i|V(r)|\psi_j\rangle \neq 0$ for $i \neq j$. Derive, to first nontrivial order in time-dependent perturbation theory, what is the probability to be in state ψ_2 as a function of time if the system is in state ψ_1 for $t < 0$.